Unit Rates + Proportions

A **unit rate** is a rate that compares the change in one quantity to a one unit change in another quantity. For example, *miles per hour* is a unit rate, because it compares the change in miles to a change of one hour. If an airplane flies 3000 miles in 5 hours, you can compute the unit rate.

It travels at $\dfrac{3000 \text{ miles}}{5 \text{ hours}} = \dfrac{600 \text{ miles}}{1 \text{ hour}}$

Solving Proportions

An equation stating that two ratios are equal is called a **proportion**. Some examples of proportions are shown at right.

$$\frac{6\text{ mi}}{2\text{ hr}} = \frac{9\text{ mi}}{3\text{ hr}}$$

$$\frac{5}{7} = \frac{50}{70}$$

When two ratios are known to be equal, setting up a proportion is one strategy for solving for an unknown part of one ratio. For example, if the ratios $\frac{9}{2}$ and $\frac{x}{16}$ are equal, setting up the proportion $\frac{x}{16} = \frac{9}{2}$ allows you to solve for x.

Strategy 1: One way to solve this proportion is by using a **Giant One** to find the equivalent ratio. In this case, since 2 times 8 is 16, so use $\frac{8}{8}$ for the Giant One.

$$\frac{x}{16} = \frac{9}{2} \cdot \boxed{\frac{8}{8}} = \frac{9 \cdot 8}{2 \cdot 8} = \frac{72}{16} \quad \text{which shows that} \quad \frac{x}{16} = \frac{72}{16} \quad \text{so } x = 72$$

Strategy 2: Undoing division. Another way to solve the proportion is to think of the ratio $\frac{x}{16}$ as, " x divided by 16." To solve for x, use the inverse operation of division, which is multiplication. Multiplying both sides of the proportional equation by 16 "undoes" the division.

$$\frac{x}{16} = \frac{9}{2}$$
$$\left(\frac{16}{1}\right)\frac{x}{16} = \frac{9}{2}\left(\frac{16}{1}\right)$$
$$x = \frac{144}{2} = 72$$

Strategy 3: Use Cross Multiplication. This is a solving strategy for proportions that is based on the process of multiplying each side of the equation by the denominators of each ratio and setting the two sides equal. It is a shortcut for using a **Fraction Buster** (multiplying each side of the equation by the denominators).

Complete Algebraic Solution (fraction buster method)	Cross Multiplication
$\dfrac{x}{16} = \dfrac{9}{2}$	$\dfrac{x}{16} = \dfrac{9}{2}$
$2 \cdot 16 \cdot \frac{x}{16} = \frac{9}{2} \cdot 2 \cdot 16$	$\frac{x}{16} \bowtie \frac{9}{2}$
$2 \cdot x = 9 \cdot 16$	$2 \cdot x = 9 \cdot 16$
$2x = 144$	$2x = 144$
$x = 72$	$x = 72$

Show the equation you are using to calculate each unit rate. Be sure to include labels.

3. Reading 258 pages in 86 minutes (pages per minute)

4. Falling 385 feet in 35 seconds (feet per second)

5. Buying 15 boxes of cereal for $39.75 ($ per box)

6. Drinking 28 bottles of water in 8 days (bottles per day)

For each proportion, show what method / calculation you used. Be clear what the solution is for X.

10. $\frac{2}{5} = \frac{x}{75}$

11. $\frac{7}{9} = \frac{14}{x}$

14. $\frac{45}{60} = \frac{x}{4}$

15. $\frac{4}{7} = \frac{18}{x}$

18. $\frac{17}{30} = \frac{51}{x}$

19. $\frac{5}{8} = \frac{16}{x}$

22. $\frac{x}{11} = \frac{8}{15}$

23. $\frac{14}{17} = \frac{x}{34}$